Lesson 13: Higher Derivatives

Warmup 1. If $f(x)$ is a function of x, find $\frac{d}{dx}[e^{f(x)}]$.

OUT=e^u

$$\frac{d}{dx}\left[e^{f(x)}\right] = e^{f(x)} \cdot f'(x)$$

We can use this to find $\frac{d}{dx}[\ln x]$:

$$\frac{d}{dx}\left[e^{\ln x}\right] = e^{\ln x} \cdot \frac{d}{dx}[\ln x]$$
$$= x \frac{d}{dx}[\ln x]$$

$$\frac{d}{dx}\left[e^{\ln x}\right] = \frac{d}{dx}[x] = 1$$

So $x \frac{d}{dx}[\ln x] = 1$

$$\frac{d}{dx}[\ln x] = \frac{1}{x}$$

2. If $f(x) = \sin x$, find:

- $f'(x) = \cos x$
- The derivative of $f'(x)$:

$$\frac{d}{dx}[\cos x] = -\sin x$$

Ex 1

$f(x) = \sin x$

$f'(x) = \cos x$ sometimes y'' or $\frac{d^2 f}{dx^2}$

$f''(x) = -\sin x$ is called the <u>second derivative</u>

$f^{(3)}(x) = \frac{d}{dx}[f''(x)] = -\cos x$ is called the <u>third derivative</u>

$f^{(4)}(x) = \frac{d}{dx}[f^{(3)}(x)] = \sin x$ is called the <u>fourth derivative</u>

In general, $f^{(n+1)}(x) = \frac{d}{dx}[f^{(n)}(x)]$.

Ex 2 $f(x) = x^2 \sin x$, find $f''(x)$.

$$f'(x) = 2x\sin x + x^2\cos x$$

$$f''(x) = \underbrace{2\sin x + 2x\cos x}_{\frac{d}{dx}[2x\sin x]} + \underbrace{2x\cos x + x^2(-\sin x)}_{\frac{d}{dx}[x^2\cos x]}$$

Acceleration $(a(t))$ is the derivative of velocity $(v(t))$. It tells us how quickly the velocity is increasing or decreasing. $a(t) = v'(t) = s''(t)$